REPORT NO. 349

COO-1469-0145

"MODEL T"
A DEMONSTRATION OF IMAGE MULTIPLICATION USING STOCHASTIC SEQUENCES

by

O. E. MARVEL

August, 1969

DEPARTMENT OF COMPUTER SCIENCE
UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN · URBANA, ILLINOIS

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*Supported in part by Atomic Energy Commision Contract No. AEC AT(11-1) 1469.

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1. INTRODUCTION

"Model T", shown in Figure 1, demonstrates a new method of electronically multiplying pictures.[1] Since the corresponding points in the two pictures are multiplied at the same time (in parallel), a large number of inexpensive multipliers must be found. Assuming that the intensity of each point on the picture can be conveniently converted to a stochastic sequence, a stochastic multiplier -- "And" gate -- costing about twenty-five cents is ideal. Since stochastic sequences represent the analog intensities of the picture, computations and processing are done with digital circuits which are fast and reliable.

Stochastic process multiplication produces with a digital "And" gate the product of two analog variables represented as the probability of occurrence of a standardized pulse occurring in a fixed time slot. When two independent stochastic pulse trains with duty cycles u_1 and u_2 are fed into an "And" gate, as shown in the figure below, the output of the "And" gate is given by

$$\begin{aligned}
u_3 &= \frac{1}{T}\int_0^T u_3(t)dt \\
&= P\{u_3(t) \text{ has a pulse in a time slot}\} \\
&= P\{u_1(t) \text{ and } u_2(t) \text{ both have pulses in a time slot}\} \\
&= P\{u_1(t) \text{ has a pulse in a time slot}\} \times \\
&\quad\; P\{u_2(t) \text{ has a pulse in a time slot}\} \\
&= u_1 \cdot u_2
\end{aligned}$$

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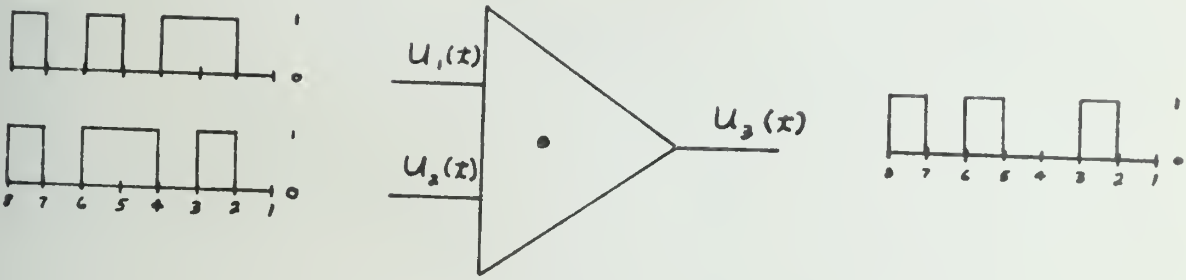
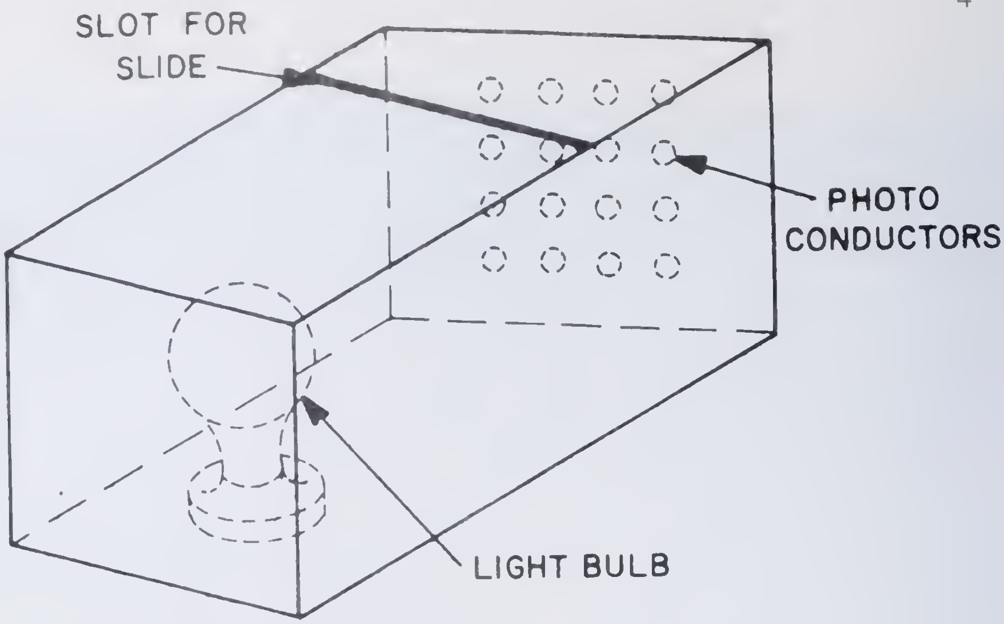
[1] This computer was developed to experimentally verify electronic circuits to be used in the TRANSFORMATRIX system and was guided by Professor W. J. Poppelbaum.

Figure 1. Model T



In "Model T", two 4 by 4 input pictures are multiplied and displayed as a 4 by 4 output picture, with 20 gray levels, on a Heathkit Model IO-18 oscilloscope, Figure 2. The output picture is produced by first converting the intensities of each input point with a photo conductor and operational amplifier to a voltage. This voltage is then compared with a Stochastic Analog Staircase (SAS) to form the Clocked Analog Stochastic Sequence (CASS) signal. Upon arrival of the proper gating pulse, the corresponding CASS signals from the two input pictures are multiplied and integrated for 32.552 microseconds. The resulting integrand, thus, represents the intensity of the output point and after being converted to an analog voltage, drives the z-axis, intensity, of the oscilloscope. Of course, at the same time the z-axis voltage arrives, the beam of the oscilloscope is correctly positioned. We have, therefore, produced a picture whose point by point brightness depends on the product of the point by point brightness from two pictures.



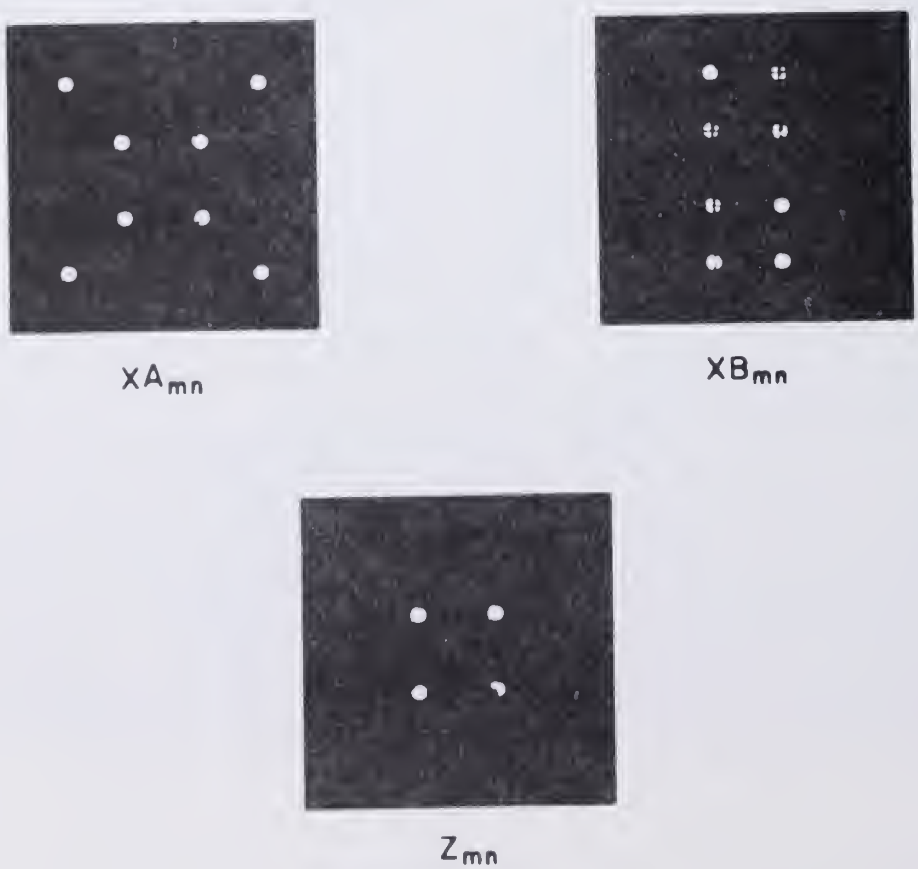
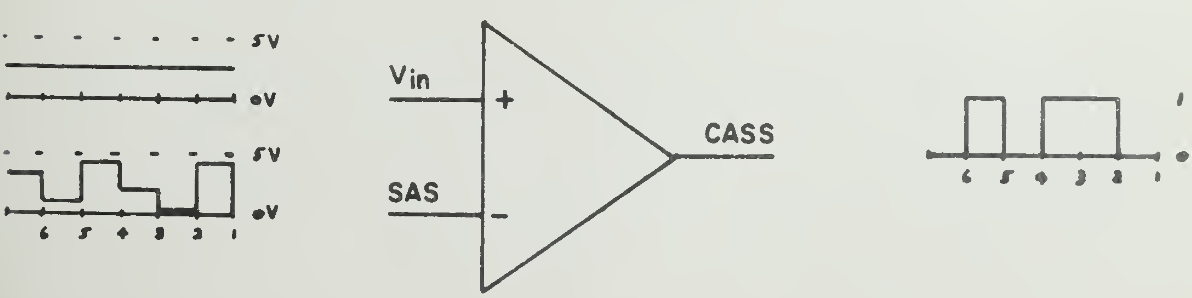


Figure 2. "Model T" Inputs

2. STOCHASTIC SEQUENCE REPRESENTATION

The probability that a pulse from a random pulse generator is present in a fixed time slot is the stochastic sequence representation of a number from 0 to 1. The above definition, although conceptually simple, is almost impossible to measure. We may, however, estimate the value of the stochastic sequence over a number of time slots. Thus, we are representing an analog variable from 0 to 1 by the number of time slots that contain a logical "1" divided by the total number of possible time slots, and this quantity may be determined by a digital counter or integrating voltmeter.

A voltage is converted to a Clocked Analog Stochastic Sequence (CASS) by comparing it with a Stochastic Analog Staircase (SAS) -- a voltage waveform that randomly takes on a number of quantized amplitude levels with equal probabilities. Assuming the voltage is between 0 to 5 volts and the SAS has 32 levels with 5/31 of a volt separating adjacent levels, then each level of the SAS has a probability of 1/32 of occurring. CASS is the output of a voltage comparitor with the inputs arranged so that when the input voltage is greater than the SAS the output is a logical 1, see below.



The accuracy of this conversion process is determined by:

(A) The voltage comparitor offset. In "Model T", a 5 millivolt worst case offset gives us 3.1% of the level separation as an error.

(B) The number of quantized levels in SAS. In "Model T", 32 levels gives us a worst case quantization accuracy of 3.1%.

(C) The statistical properties of the Pseudo-Random-Noise Generator. "Model T" uses a 34 bit feedback shift register which has a worst case probability error which is very small.

After the stochastic sequences have been processed, they are converted into a voltage by an averaging circuit. The accuracy of this conversion process can be obtained from the probability theory of stochastic sequences. If the probability of a pulse occurring in a time slot is p, then the probability of no pulse is 1 - p. The mean or expected value of the stochastic sequence is obtained from:

$$\bar{m} = E(x) = \Sigma\{(\text{probability of value})(\text{value})\}$$

$$= (p)(1) + (1-p)(0) = p$$

and the variance obtained from:

$$\sigma^2 = E[(x-m)^2] = E(x^2) - [E(x)]^2 = E(x^2) - m^2$$

$$\sigma^2 = p(1)^2 + (1-p)(0)^2 - p^2 = p - p^2 = p(1-p)$$

Since there are N possible time slots, the mean is Np and the variance is Np(1-p).

The accuracy to be specified here is a percentage of full scale reading as in a voltmeter. Since each output point is calculated and displayed 30 times per second, while the human eye and oscilloscope phosphor act as averagers, a 95% confidence level per display seems reasonable. Thus the 2σ standard deviation error is given by

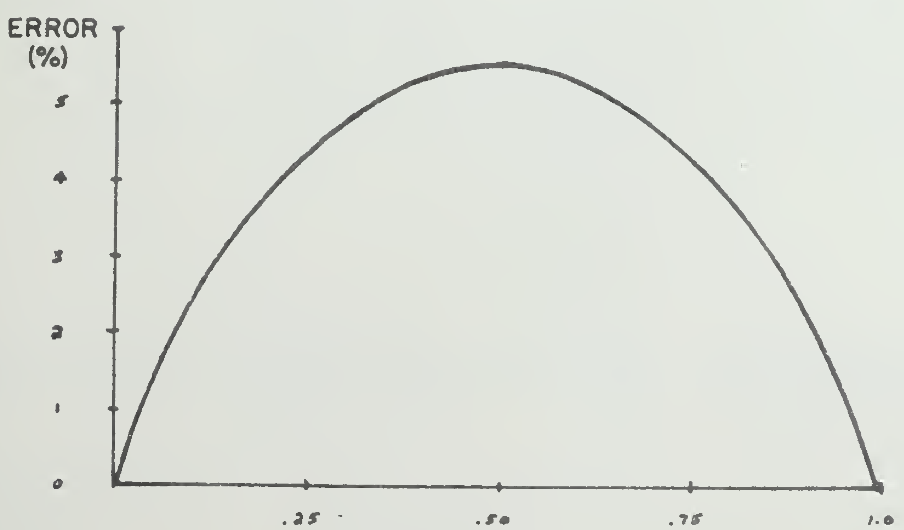
$$\text{Error} = \frac{2\sigma}{N} \times 100\%$$

$$= \frac{200}{N}[Np(1-p)]^{\frac{1}{2}}$$

$$= \frac{200}{(N)^{\frac{1}{2}}}[p(1-p)]^{\frac{1}{2}}$$

For "Model T" N is 325 giving

$$\text{Error} = 11.1[p(1-p)]^{\frac{1}{2}}$$



3. TIMING

The basic timing is dictated by the following three requirements.

(A) The output display frame rate must be the same as the frame rate of a conventional TV receiver[2] - 30 frames/second.

(B) The fastest economical digital circuits in integrated circuit form are transistor-transistor logic circuits with a maximum practical pulse repetition rate of ten megahertz.

(C) The circuits when proven in "Model T" will be used in the TRANSFORMATRIX system which will display a 32 x 32 output picture.

Since TRANSFORMATRIX will display 1,024 output points at a rate of 30 frames/second, a new output point will be calculated every 32.552 microseconds or at a rate of 30.72 kilohertz. This gives us a master clock frequency of 9.984 megahertz and 325 time slots per stochastic process.

Figure 3 shows the timing and gating pulses which are derived from a 9.984 megahertz multivibrator -- Master Clock. A counter then produces an output pulse -- Point Clock -- for every 325 Master Clock pulses.

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[2] In this display there is no interlacing, thus the frame and field rates are equal.

Since "Model T" only displays 16 intensities per frame, it processes data for only .52 milliseconds while it rests for 32.81 milliseconds every frame. A counter selecting 16 of the point clock pulses out of every 1024 produces the Input Clock. Monostable multivibrators are used to produce Shift Clock, Reset Clock, and the blanking and rest pulses from Input Clock.

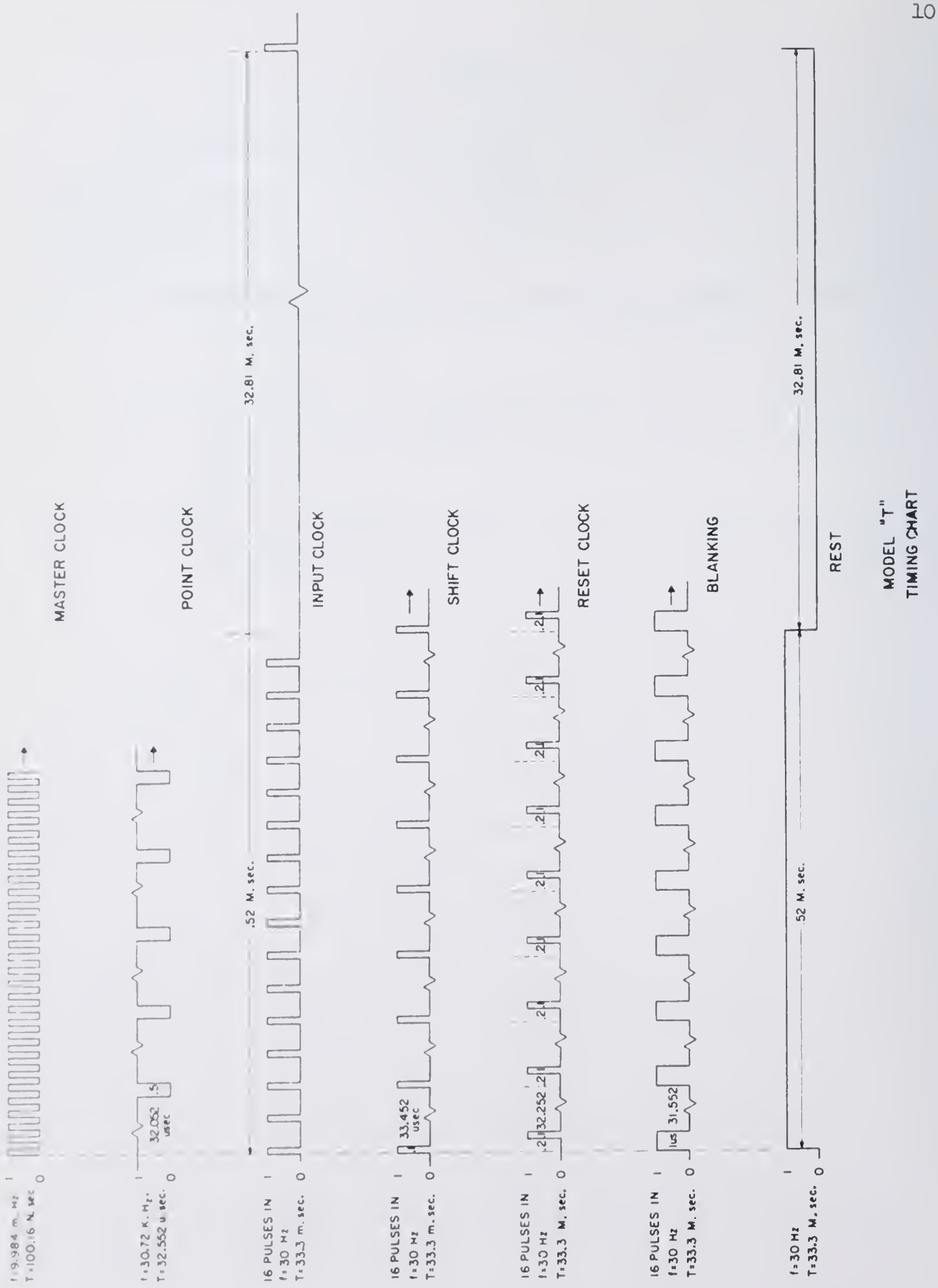

Figure 3. Timing Chart

4. SYSTEM OPERATION

The block diagram for "Model T" is shown in Figure 4. The basic timing and gating pulses as discussed in the last section are produced in the clock control and gating control units. Because the oscilloscope is displaying the intensity of an output point while the next output points' intensity is being calculated, the 4 bit X-Y counter must produce the present X-Y position as X and Y analog voltages for the oscilloscope and the next X-Y position as digital commands for multiplexing the CASS signals.

A 34 bit feedback shift register produces two 5 bit digital noise signals which are converted to the Stochastic Analog Staircase - SAS - by digital to analog converters. Since the two digital noise signals are independent, the two SAS signals are independent also. Each SAS goes to a set of sixteen comparitors whose other inputs are the voltages representing the light intensities from the two input pictures. Thus, we are generating sixteen pairs of stochastic sequences, each pair representing one of the input pictures.

The input processors shown in Figure 4 perform two operations on the stochastic sequences. First, the independent stochastic sequences, CASS, representing the same grid location on each picture must be multiplied. Then, because the oscilloscope only displays one product at a time as an output intensity, the 16 CASS products must be multiplexed or funneled one at a time to the integrator.

The integrator produces a binary representation of the output intensity by integrating for 32.552 microseconds the CASS. During the next 32.552 microseconds period, the binary number is stored and connected to a voltage which is applied to the z axis of the oscilloscope through the driver switching unit. Two other signals are applied to the driver switching circuit: The first is a blanking pulse which turns off the beam during the time it is moving from point to point. The second is the rest pulse which cuts off the beam during the 32.81 millisecond rest period.

The driver switching unit allows either the positive or negative product of the input pictures to be displayed.

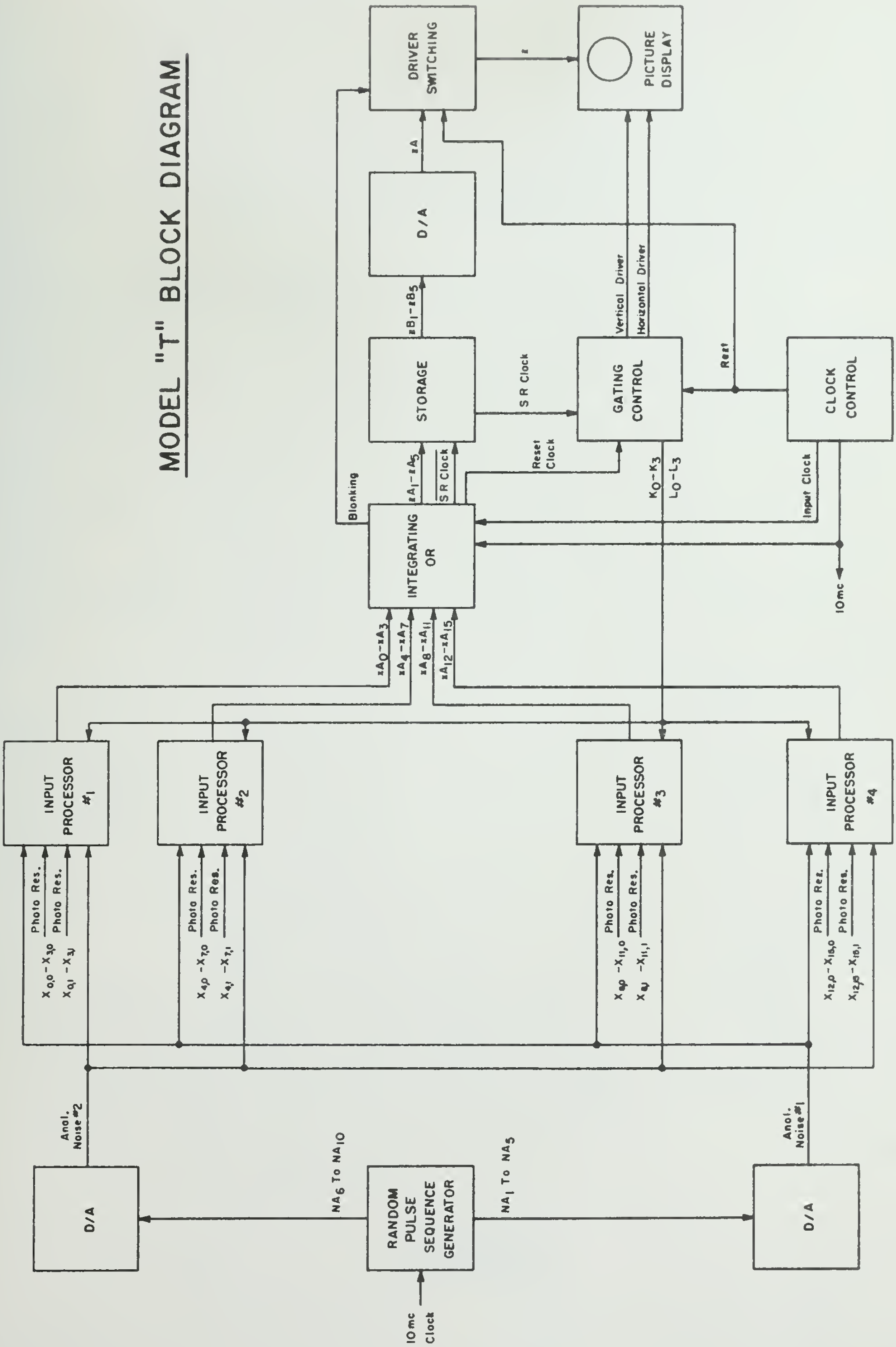

Figure 4. Block Diagram of "Model T"

5. CIRCUITS

To keep cost and size down, integrated circuits are used throughout "Model T". All digital circuits are positive logic TTL integrated circuits.

Figure 5 shows the syncronous random pulse generator and the two stochastic analog staircase - SAS - generators. The syncronous random pulse generator is a 34 bit feedback shift register where the bit to be entered and shifted is given by $Q_0 = Q_{34} \oplus Q_{33} \oplus Q_{32} \oplus Q_7$. Thus, the shift register contains a continuously changing - at a 10 MHz rate - binary word in which the probability of any bit being a one is 1/2. Also, at any instant of time, i.e. within a fixed word, the value of any bit is independent of the value of any other bit.

The outputs of 5 stages of the shift register go to one high speed D/A converter to produce SAS #1, and the outputs of 5 different stages go to another D/A converter to produce SAS #2. The digital to analog converter consists of a high speed voltage switch with weighted resistor decoder and high speed operational amplifier.

Figure 6 shows 16 identical input processor units which perform four functions. First the light intensity to voltage transfer function must be standardized. The transfer function for the photoconductor is given by

$$R = R_0(I)^{-.84}$$

where R is the resistance in ohms, R_0 is the resistance at one foot candle which may vary $\pm$ 33%, and I is the intensity in foot candles.

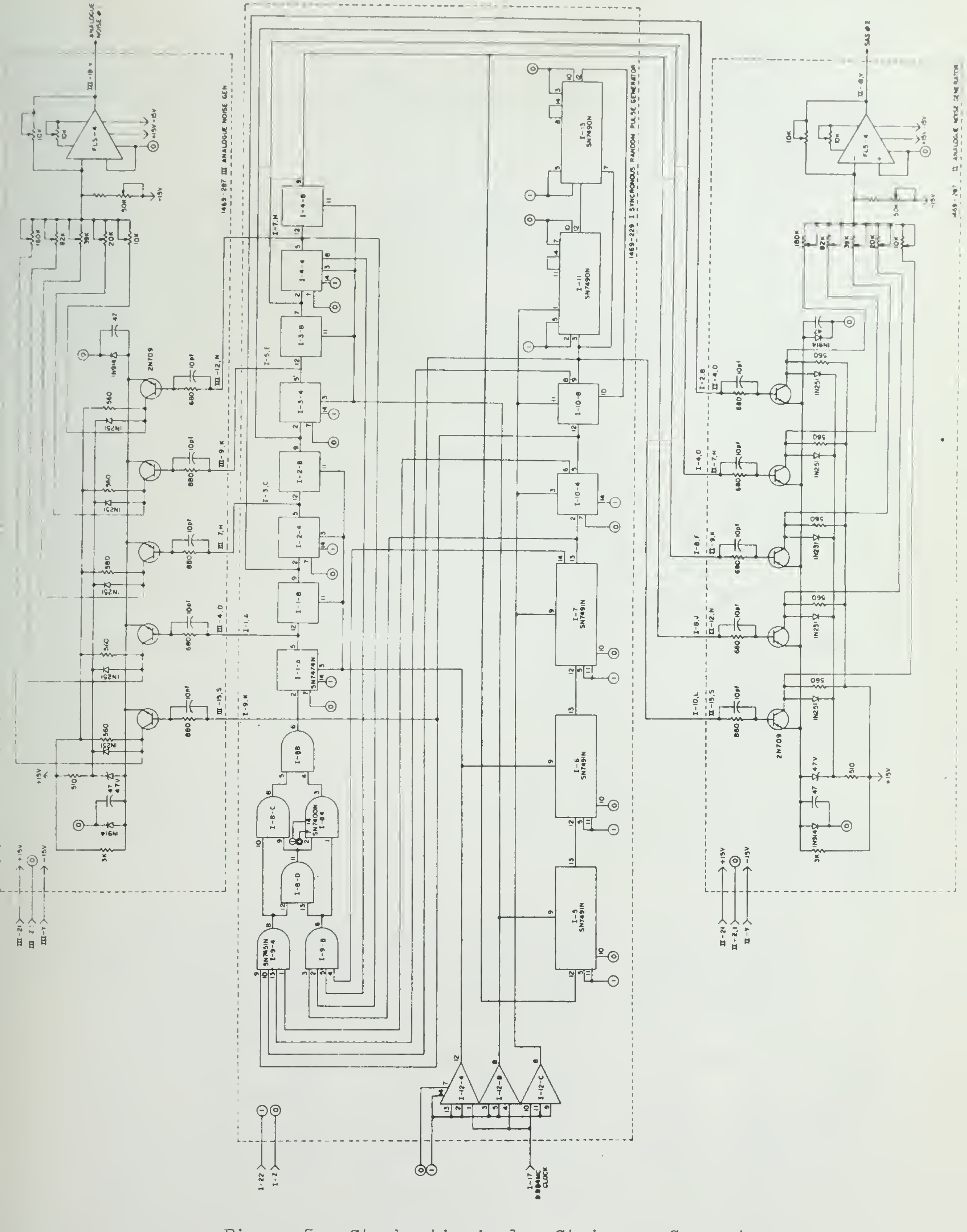

Figure 5. Stochastic Analog Staircase Generator

With the photo conductor as the input resistor and a trimpot R_F as the feedback resistor of an operational amplifier, the output voltage of the op amp is

$$V_{INT} = \frac{R_F}{R_O}(I)^{+.84} V_S$$

Thus with V_S a standardized voltage, the variation in R_O can be cancelled by adjusting R_F to make all photo processors have the same transfer functions.

The CASS signals are produced by comparing the V_{INT}s against the SAS signal with a 710 voltage comparitor which produces a logical one whenever V_{INT} is greater than the SAS signal. Since the comparitors have a maximum differential input voltage of 5 volts and the SAS has 32 levels, to obtain the greatest noise margin the photo processor stage (R_F) is adjusted to give V_{INT} a 5 volt swing as the light varies from .1 to 1.0 foot candles.

A four input NAND gate, when enabled by the proper I and J gating signals, produces the logical inverse of the product of two CASS input signals.

Figure 7 shows the output intensity processing circuitry which performs 16 identical cycles per displayed frame. After the 16 lines from the input processor have been inverted and OR'ed, the CASS product is AND'ed with a 9.984 MHz square wave to produce a pulse when CASS is a logical one, except for three periods when the AND gate is inhibited and no pulses are produced. A ripple counter then counts the number of 9.984 Mhz pulses in a 32.552 microsecond period. At the end of this period, while the input to the counter is inhibited, the 5 MSB's of the

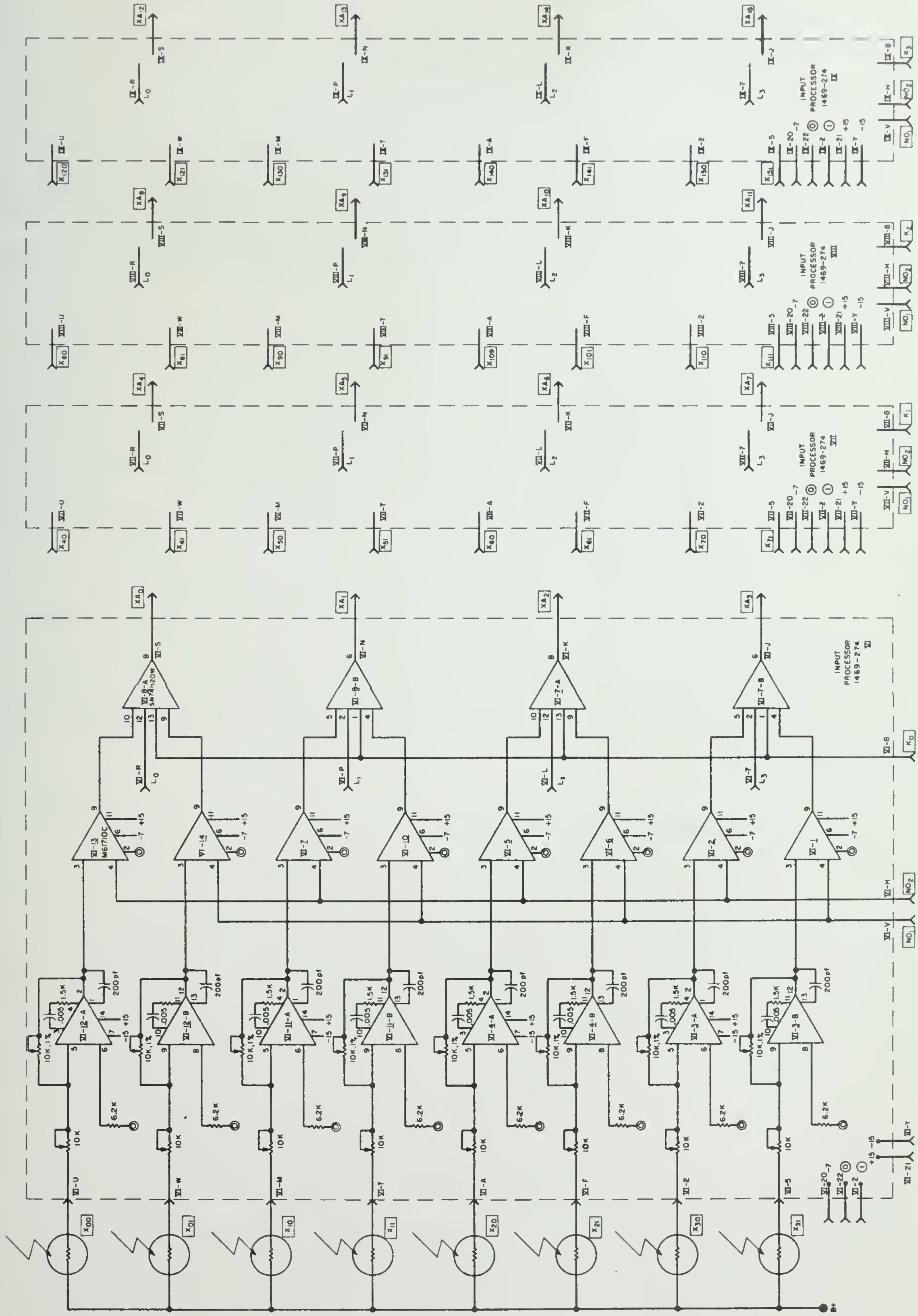

Figure 6. Input Processors

counter are shifted into a 5 bit storage register and the counter reset. During the next cycle the binary number in the storage register is converted to an analog voltage by a D/A converter. This analog voltage is applied to the z axis of our oscilloscope through a $\pm$ 25 volt driver stage which has the ability to amplify the applied voltage - display the picture - or invert and amplify the applied voltage - display the negative of the picture.

Simultaneous with the signal flow shown in the last three figures, timing and gating signals are generated as shown schematically in Figure 8. Master clock is a 9601 multivibrator wired as a 9.984 MHz astable multivibrator. Ripple counters and coincidence circuits are used to produce point clock and input clock.

The 2 bit X and Y counters, Figure 8, are decoded to produce the I_0 to I_3 and J_0 to J_3 gating signals. The outputs of the counters are also delayed one cycle and converted to M and N, the horizontal and vertical drivers.

Two problems were encountered and solved after "Model T" was built. The first was the long rest time built into "Model T" which caused a bright spot on the oscilloscope at X_0,Y_0. This problem was eliminated by connecting a retriggerable monostable multivibrator, with its period set slightly longer than the period of input clock - 32.552 microseconds, to input clock. Thus the inverted output of the multivibrator is zero while we have the 16 pulses of input clock and +4 volts which is used to blank the oscilloscope when there are no input clock pulses. The second problem was the capacitive coupling of the z axis which causes intensity level standardization problems. This problem

will be eliminated in TRANSFORMATRIX by using an oscilloscope with DC coupling. But, in "Model T", the impedence of the grid circuit of the oscilloscope tube as well as the input capacitance were increased, giving a more satisfactory time constant for the z axis grid drive.

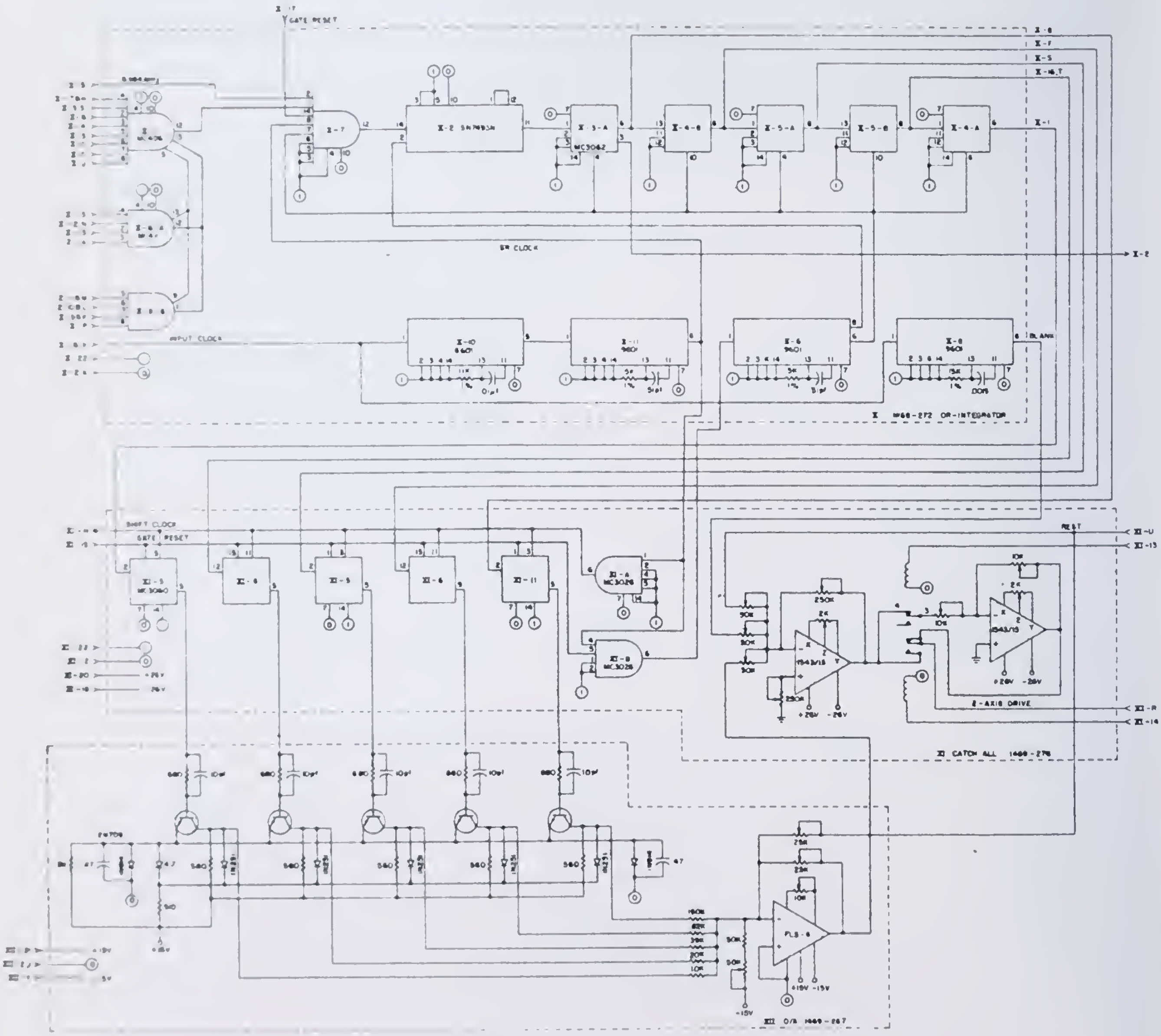

Figure 7. Output Processor

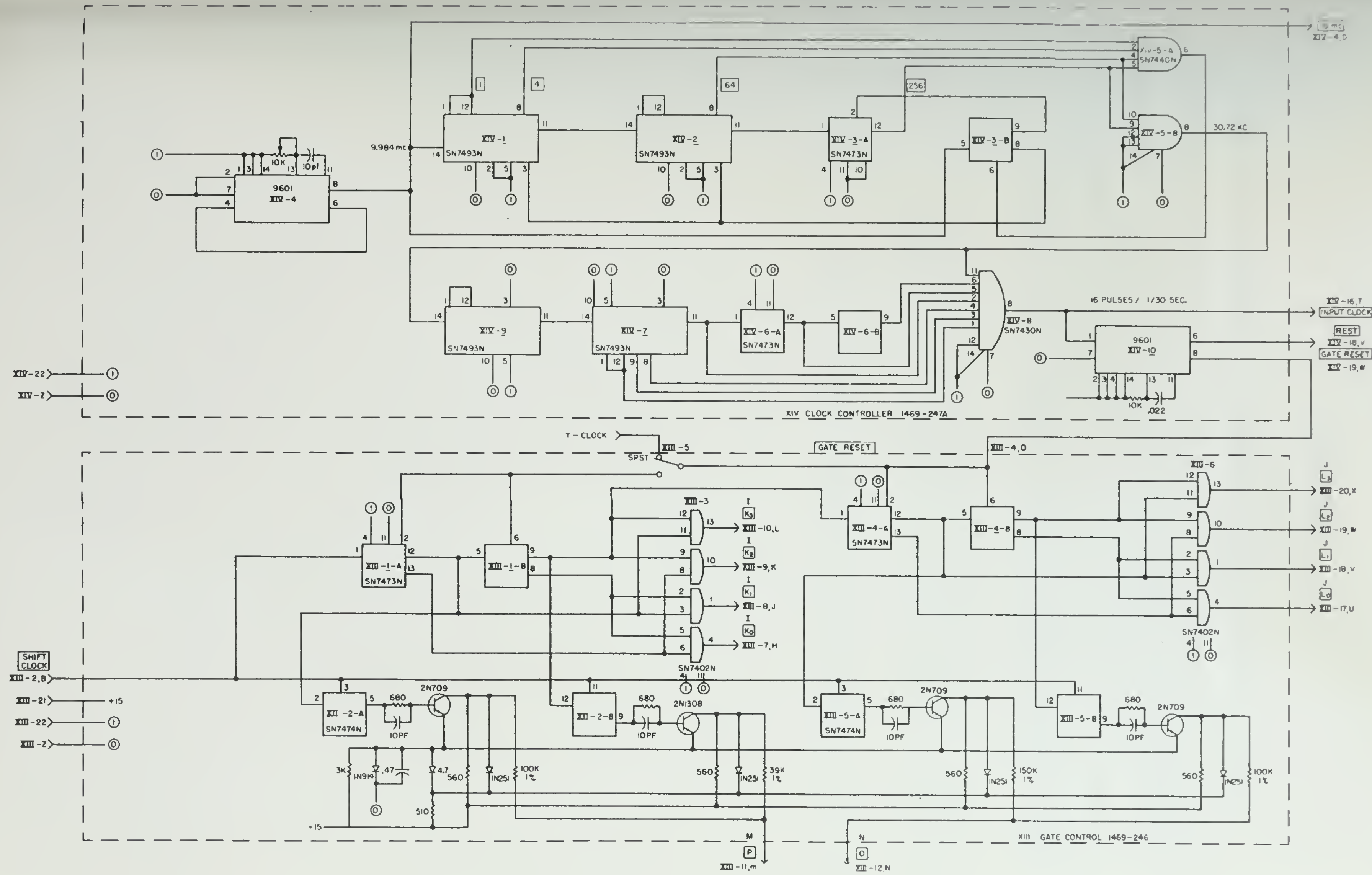

Figure 8. Timing and Gating Generation

6. CONCLUSIONS

Because of the ease and low expense of multiplying and gating analog quantities in stochastic sequence form, large parallel processors may be economically built using stochastic elements. "Model T" by using these stochastic techniques has shown the feasibility and problems that must be overcome to process pictures at standard television rates.

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